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Lagrangian Relaxation and Tabu Search Approaches for the Unit Commitment Problem

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Outline of presentation

1. Problem formulation

2. Different computational methods

   3.1 Lagrangian relaxation

   3.2 Tabu search

3. Numerical results

4. Conclusions

Problem formulation

Determine
which electrical power generators to commit
their production levels

to supply
forecasted short-term (24 –168 hours) demand
spinning reserve requirements
at minimum cost

thermal units: operating cost, history-dependent startup costs, discrete on/off decision, minimum up/down-time, ramping constraints, emission characteristics

Problem formulation

Cost function

- Production cost

$$C_i(P_{it}) = a_{0i} + a_{1i}P_{it} + a_{2i}P_{it}^{\;2}$$

$$\min_{\mathbf{u},\mathbf{p}} \left\{ \sum_{i=1}^{I} \sum_{t=1}^{T} \left[c_i\left(u_{i,t}, p_{i,t}\right) + s_i\left(u_{i,t-1}, u_{i,t} \right) \right] \right\} = \min_{\mathbf{u},\mathbf{p}} C\left(\mathbf{u},\mathbf{p}\right)$$

- Start-up cost

System constraints

- Load demand

$$D_t - \sum_{i=1}^{I} p_{i,t} = 0 \qquad \forall t = 1,\ldots,T$$

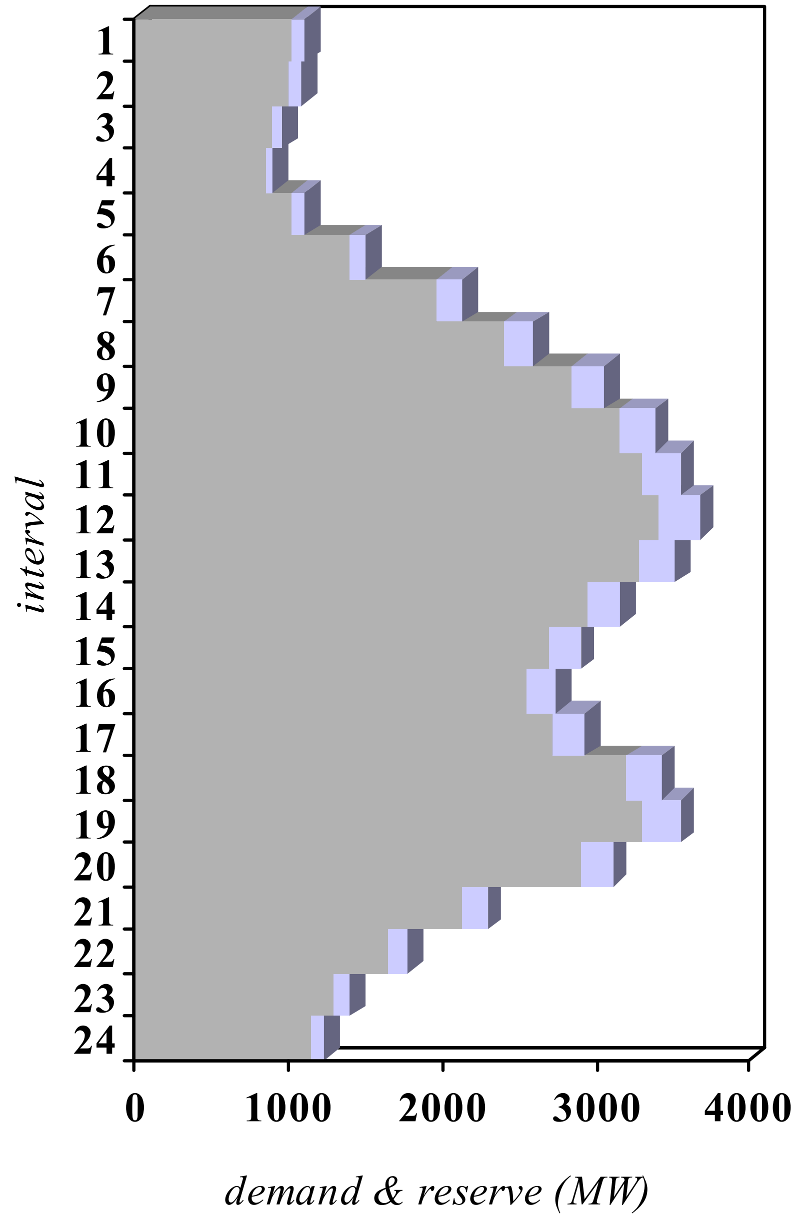
Unit constraints

- Generation limits
- Minimum up/down time
- Unit initial status

$$\left.\begin{array}{l} u_{i,t} \cdot p_i^{\min} \le p_{i,t} \le u_{i,t} \cdot p_i^{\max} \\ \tau_i^d \text{ and } \tau_i^u \text{ constraints} \end{array}\right\} \quad \begin{array}{l} \forall i = 1,\ldots,I \\ \forall t = 1,\ldots,T \end{array}$$

Problem formulation

- The study period is divided into smaller time intervals of equal duration
- The load is assumed to be constant within each time period
- The typical duration that is considered for every division is one hour
- Transitions between commitment states (ON/OFF) of generating units are allowed only at the beginning of each interval



Lagrangian relaxation

Relaxing global constraints (demand requirements) yields the following dual approach:

$$L(\boldsymbol{\lambda}) = \min_{\mathbf{u},\mathbf{p}} \left\{ C(\mathbf{u},\mathbf{p}) + \sum_{t=1}^{T} \lambda_t \cdot \left(D_t - \sum_{i=1}^{I} p_{i,t} \right) \right\}$$

subject to

$$\left.\begin{array}{c} u_{i,t} \cdot p_i^{\min} \le p_{i,t} \le u_{i,t} \cdot p_i^{\max} \\ \tau_i^d \quad \text{and} \quad \tau_i^u \quad \text{constraints} \end{array}\right\} \quad \begin{array}{l} \forall i = 1,\ldots,I \\ \forall t = 1,\ldots,T \end{array}$$

Lagrangian relaxation

The dual function is rearranged as

$$L(\boldsymbol{\lambda}) = \sum_{i=1}^{I} L_i(\boldsymbol{\lambda}) + \sum_{t=1}^{T} \left(\lambda_t \cdot D_t\right)$$

where, for unit *i*

$$L_i(\boldsymbol{\lambda}) = \min_{\mathbf{u}_i, \mathbf{p}_i} \left\{ \sum_{t=1}^{T} \left[c_i(u_{i,t}, p_{i,t}) - \lambda_t \cdot p_{i,t} + s_i(u_{i,t-1}, u_{i,t}) \right] \right\}$$

Lagrangian relaxation

Solution in two steps

1) Assigned λ and $\mathbf{u}_i$, $\mathbf{p}_i$ is found by solving

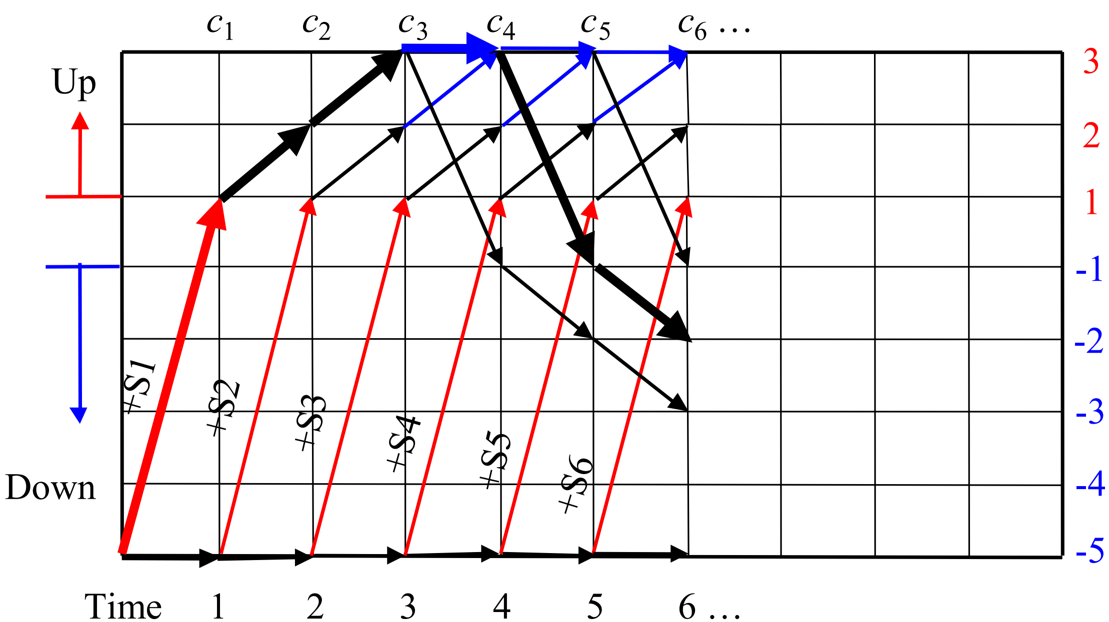
$$\tilde{L}_i(\boldsymbol{\lambda}) = \min_{\mathbf{p}_i} \left\{ \sum_{t=1}^{T} \left[c_i(u_{i,t}, p_{i,t}) - \lambda_t \cdot p_{i,t} \right] \right\}$$

subject to

$$u_{i,t} \cdot p_i^{\min} \leq p_{i,t} \leq u_{i,t} \cdot p_i^{\max} \quad \forall t = 1, \ldots, T$$

Lagrangian relaxation

2) Assigned $\mathbf{p}_i$, $\mathbf{u}_i$ is found by means of a forward dynamic programming algorithm



subject to

$$\tau_i^d \quad \text{and} \quad \tau_i^u \quad \text{constraints}$$

Lagrangian relaxation

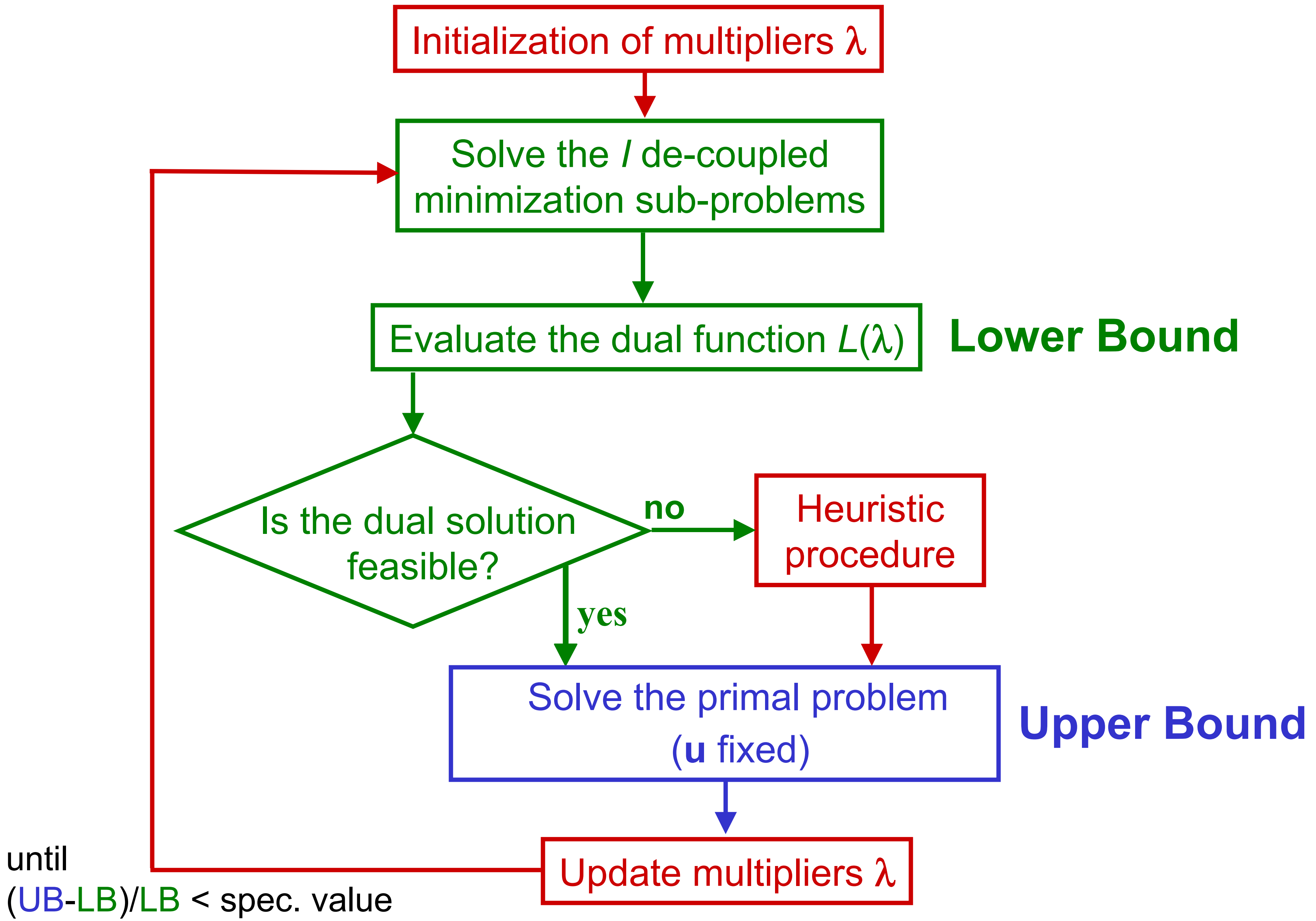
To find λ, the following dual problem is solved

$$L^* = \max_{\lambda \geq 0} L(\boldsymbol{\lambda})$$

L* is a lower bound on the optimal objective value of the primal problem.

Critical points:

1) how to solve the dual problem
2) how to compute a feasible solution


Initialization of multipliers λ
Solve the I de-coupled minimization sub-problems
Evaluate the dual function L(λ)
Lower Bound
Is the dual solution feasible?
no
yes
Heuristic procedure
Solve the primal problem (u fixed)
Upper Bound
Update multipliers λ
until
(UB-LB)/LB < spec. value

Lagrangian relaxation

Initialization of λ

Solve a continuous ($u_{it} \in [0, 1]$) *relaxed* version of the primal problem

$$\min_{\mathbf{u},\mathbf{p}} \left\{ \sum_{i=1}^{I} \sum_{t=1}^{T} \left[c_i \left(u_{i,t}, p_{i,t} \right) + s_i \left(u_{i,t-1}, u_{i,t} \right) \right] \right\} = \min_{\mathbf{u},\mathbf{p}} C(\mathbf{u},\mathbf{p})$$

$$D_t - \sum_{i=1}^{I} p_{i,t} = 0 \qquad \forall t = 1,\ldots,T$$

$$\left. \begin{array}{ll} p_{i,t} \le p_i^{\max} & \\ u_{i,\min(T,t+r)} \ge u_{i,t} - u_{i,t-1} & \forall r = 1 \ldots \tau_i^{up} \\ u_{i,\min(T,t+r)} \le 1 - u_{i,t-1} + u_{i,t} & \forall r = 1 \ldots \tau_i^{d} \end{array} \right\} \quad \begin{array}{l} \forall i = 1,\ldots,I \\ \forall t = 1,\ldots,T \end{array}$$

Lagrangian relaxation

Solution of the Lagrangian Dual

$$L^* = \max_{\boldsymbol{\lambda} \geq 0} \; L(\boldsymbol{\lambda})$$

The subgradient $\mathbf{g}(\boldsymbol{\lambda})$ of $L(\boldsymbol{\lambda})$ with respect to Lagrangian multipliers $\boldsymbol{\lambda}$ is a T-vector. The t-th element is

$$g_t(\boldsymbol{\lambda}) = D_t - \sum_{i=1}^{I} p_{i,t}$$

At iteration k, the bundle method accumulates multipliers $\boldsymbol{\lambda}_1, \ldots, \boldsymbol{\lambda}_k$, subgradients $\mathbf{g}(\boldsymbol{\lambda}_1), \ldots, \mathbf{g}(\boldsymbol{\lambda}_k)$ and dual function values $L(\boldsymbol{\lambda}_1), \ldots, L(\boldsymbol{\lambda}_k)$ in a *bundle* $< \boldsymbol{\lambda}_k, \mathbf{g}(\boldsymbol{\lambda}_k), L(\boldsymbol{\lambda}_k) >$. With this bundle, $L(\boldsymbol{\lambda})$ is upper approximated with the following *cutting plane (CP) model*

$$L_k^{\mathrm{CP}}(\boldsymbol{\lambda}) = \min_{1 \leq j \leq k} [L(\boldsymbol{\lambda}_j) + \mathbf{g}(\boldsymbol{\lambda}_j)' \cdot (\boldsymbol{\lambda} - \boldsymbol{\lambda}_j)]$$

Lagrangian relaxation

At iteration k

$\bar{\boldsymbol{\lambda}}$ is the *current point*, i.e. the T-vector of $\boldsymbol{\lambda}$ values yielding the highest LB currently available

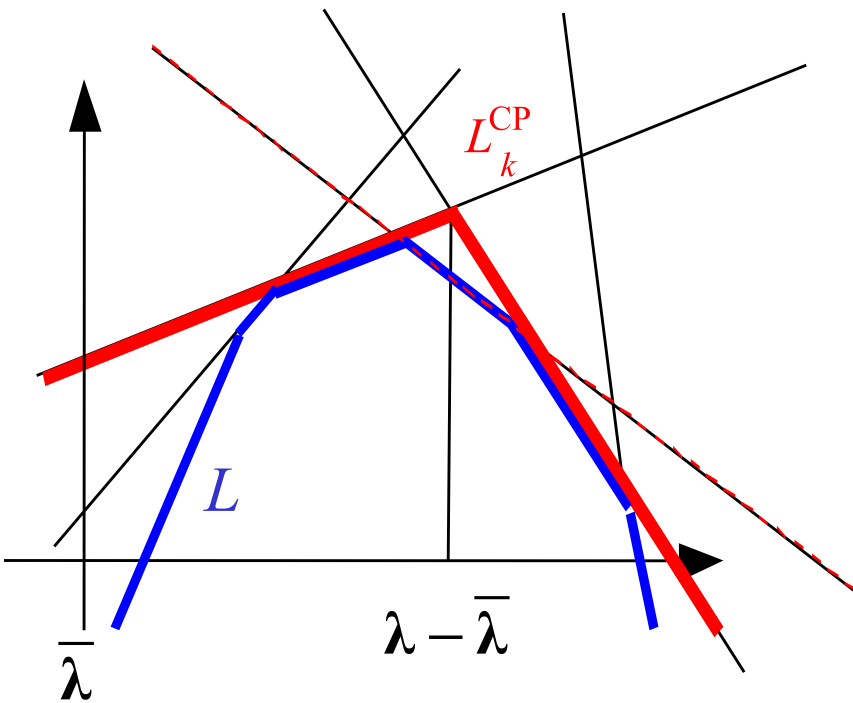
$$\Delta L_j = L(\boldsymbol{\lambda}_j) + \mathbf{g}(\boldsymbol{\lambda}_j)' \cdot (\bar{\boldsymbol{\lambda}} - \boldsymbol{\lambda}_j) - L(\bar{\boldsymbol{\lambda}})$$

then λ_{k+1} is obtained from

$$\max_{\boldsymbol{\lambda}, v} \; v$$

$$\text{subject to } v \le \Delta L_j + \mathbf{g}(\boldsymbol{\lambda}_j)' \cdot (\boldsymbol{\lambda} - \bar{\boldsymbol{\lambda}})$$

$$\forall 1 \le j \le k$$

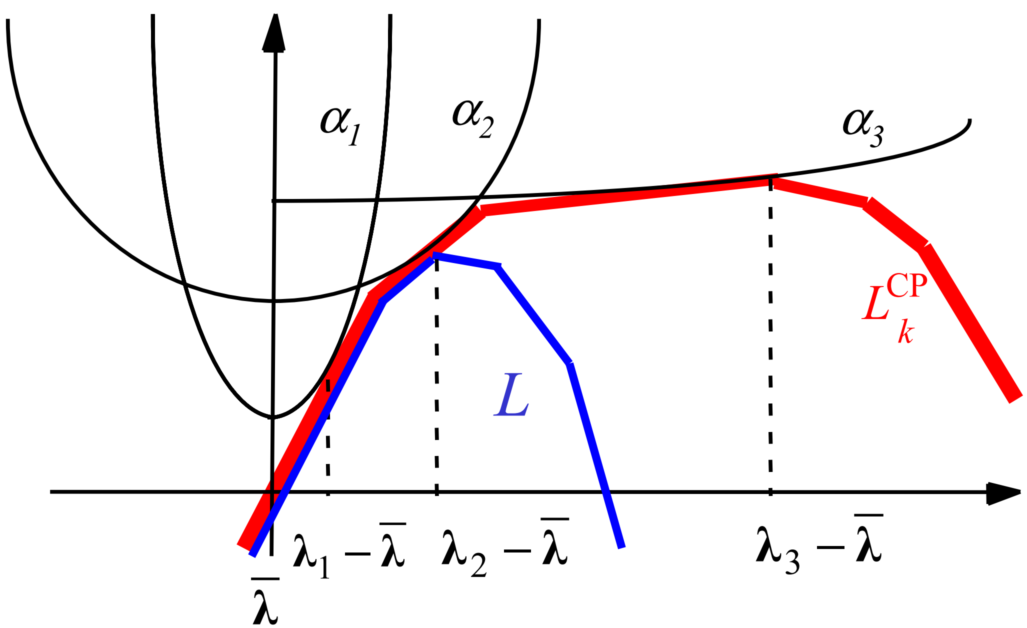


Lagrangian relaxation

Stabilized solution

$$\max_{\boldsymbol{\lambda},v} \; [v - \frac{1}{2\cdot\alpha}\left\|\boldsymbol{\lambda}-\overline{\boldsymbol{\lambda}}\right\|]$$

subject to $v \le \Delta L_j + \mathbf{g}(\boldsymbol{\lambda}_j)' \cdot (\boldsymbol{\lambda}-\overline{\boldsymbol{\lambda}}) \qquad \forall 1 \le j \le k$



$\alpha_1 < \alpha_2 < \alpha_3$

$\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2, \boldsymbol{\lambda}_3$ optimal solutions

α *trust region parameter*, ie how far from $\overline{\boldsymbol{\lambda}}$ L^{CP} is believable

Bundle disaggregato

Lagrangian relaxation

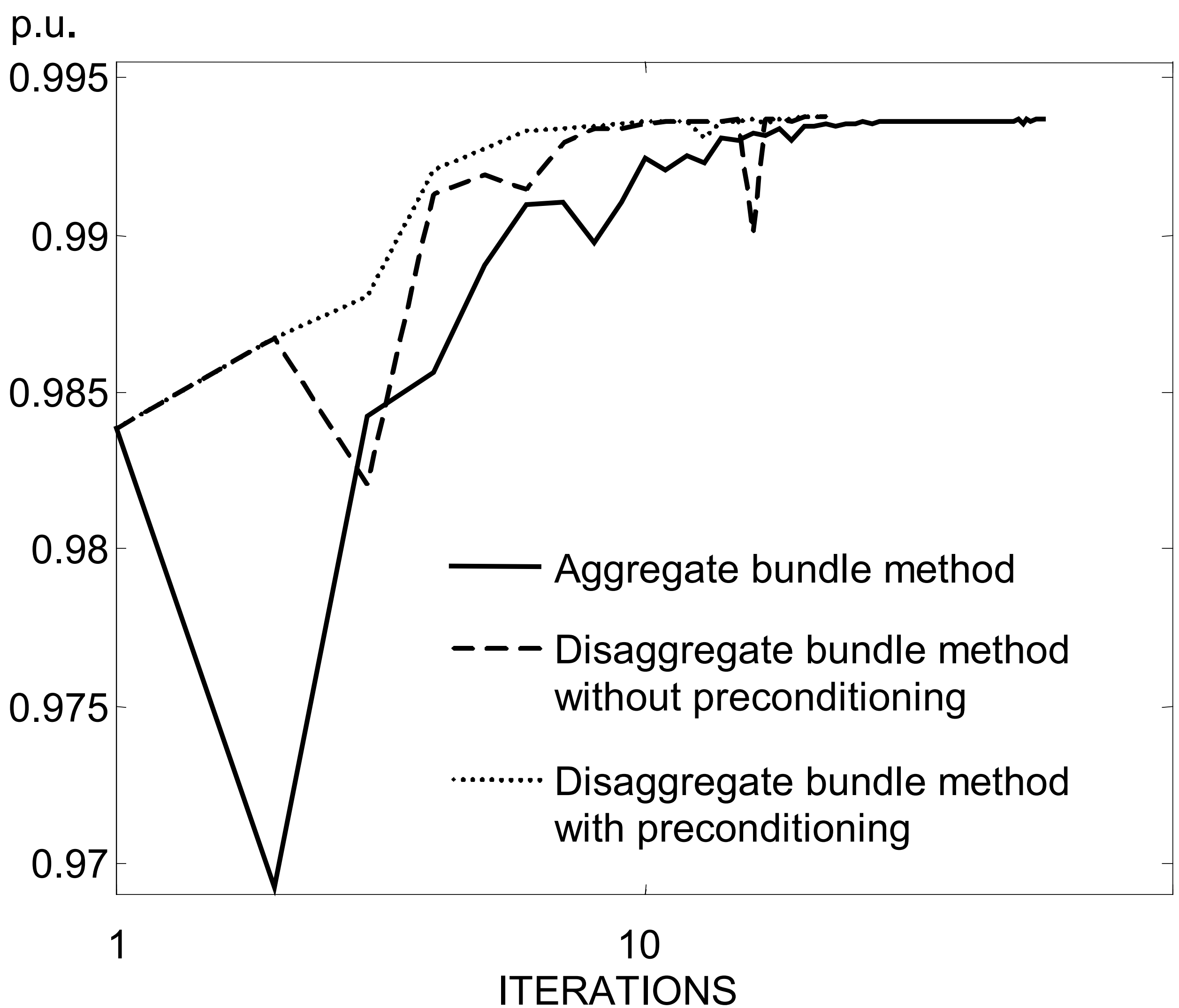
The Heuristics

By solving the dual problem by a bundle method, without an extra computational effort, a “convexified” solution of the original problem is also available. This solution is a matrix with the same dimensions of matrix **u**, whose elements $u_{i,t} \in [0,1]$ can be interpreted as the “probability” for unit *i* to be committed at period *t*.

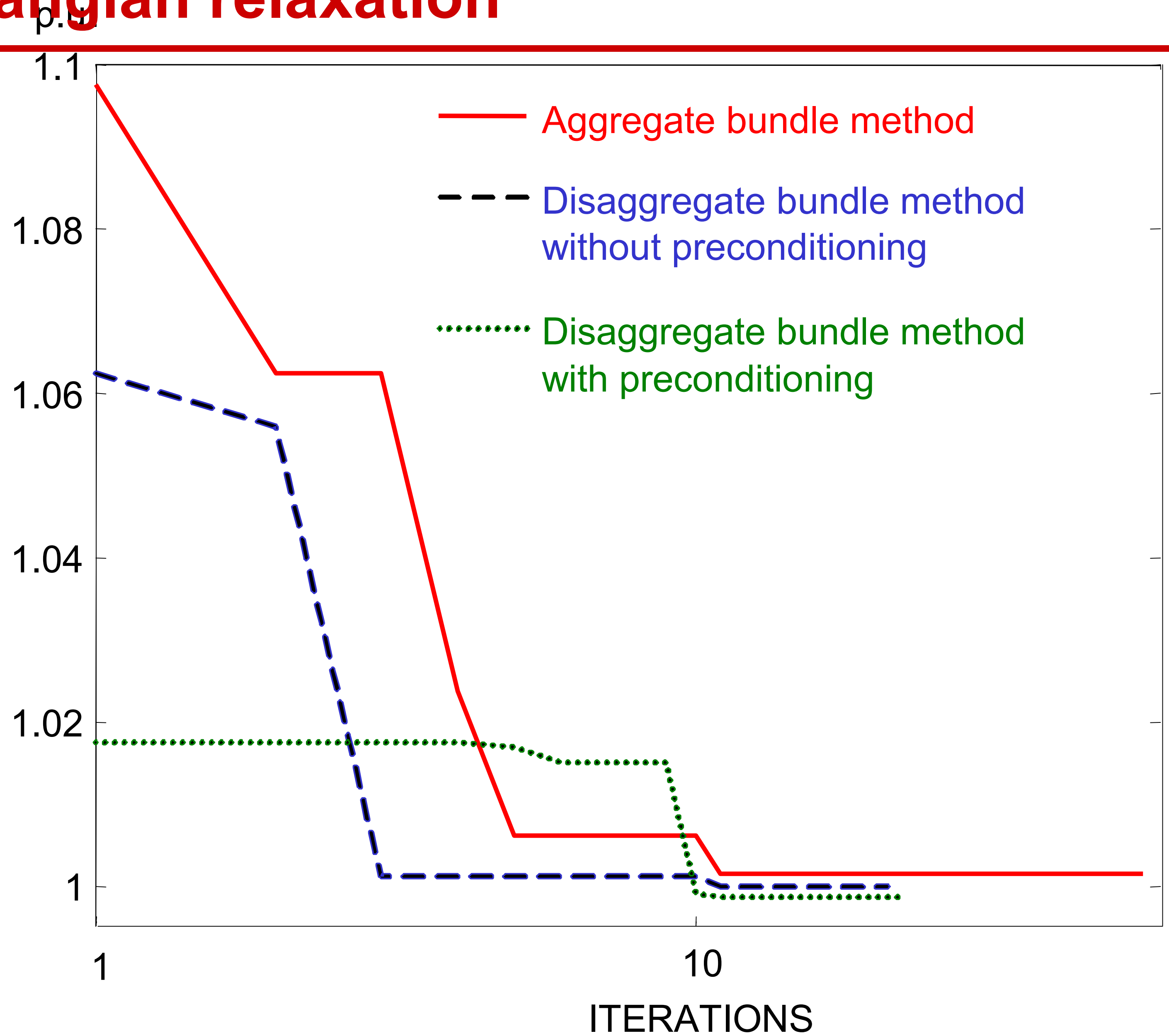
Making use of this matrix, a heuristic procedure has been implemented trying to uncommit units that are not really needed.

Such a heuristic procedure results in significantly improving the overall performance of the algorithm.

Lagrangian relaxation

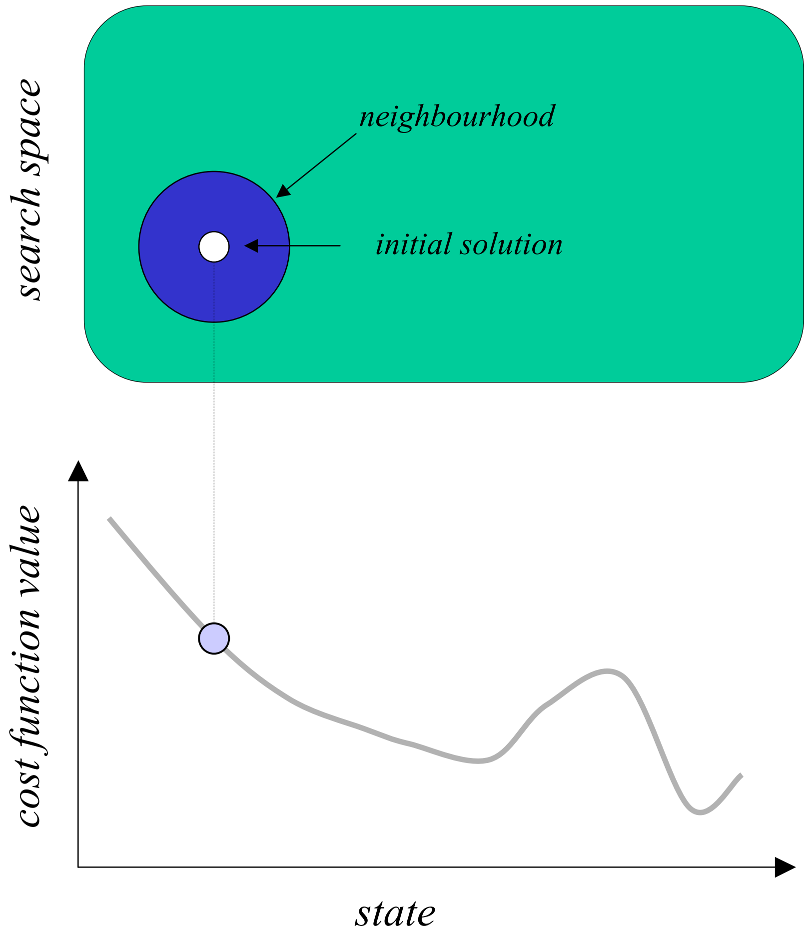


Lagrangian relaxation



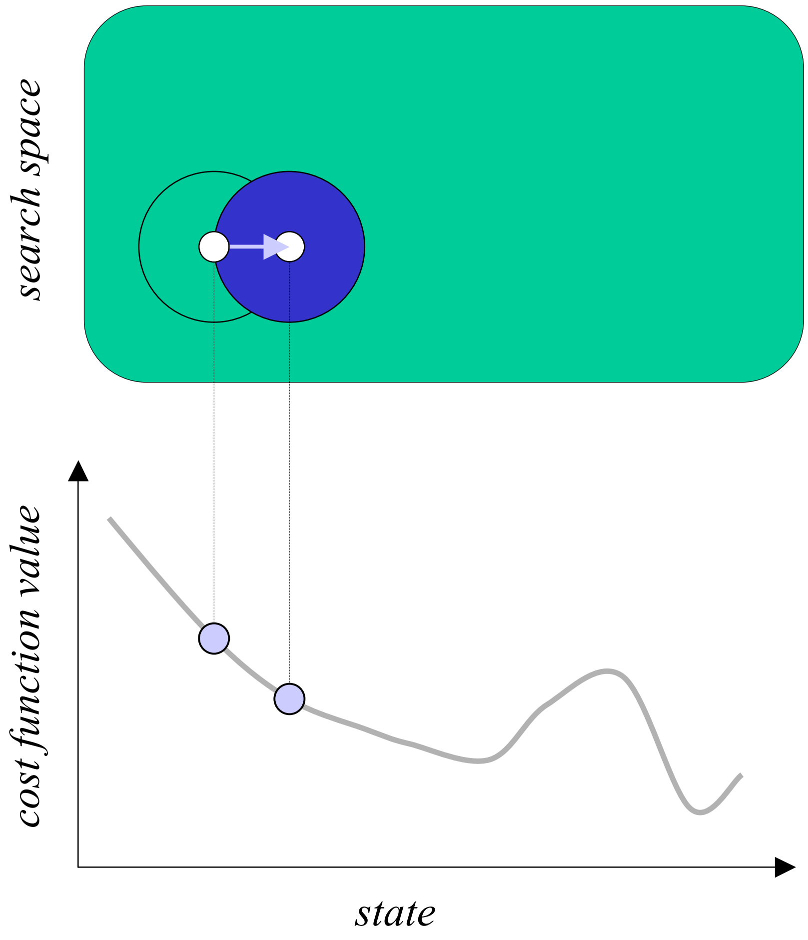
Tabu search

Neighbourhood search in TS



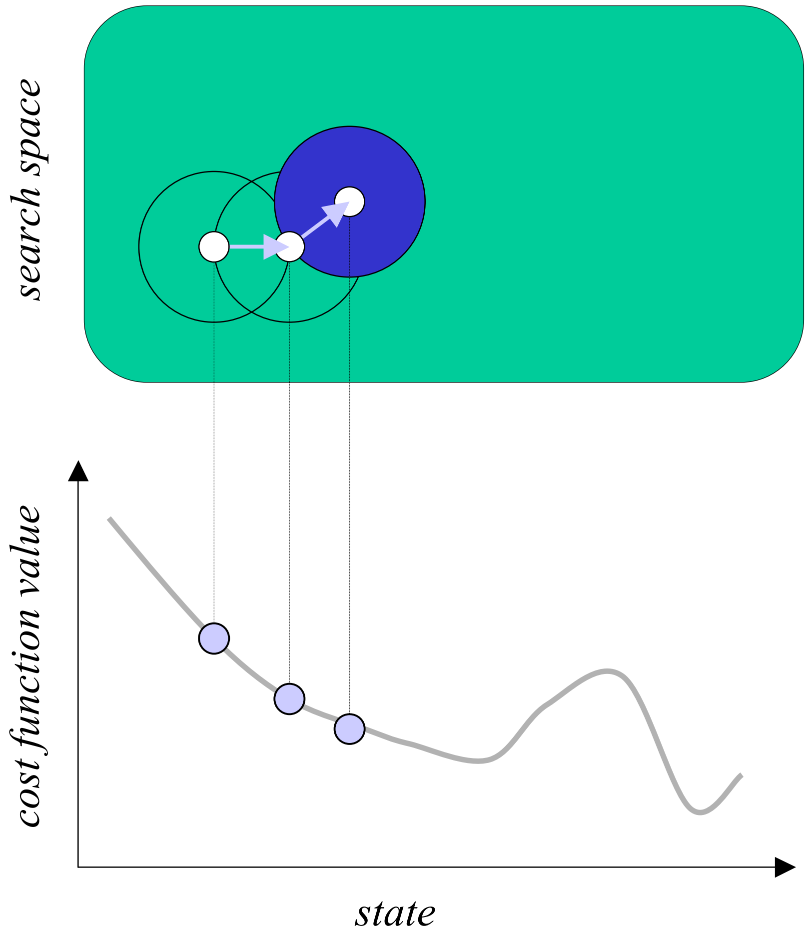
Tabu search

Neighbourhood search in TS



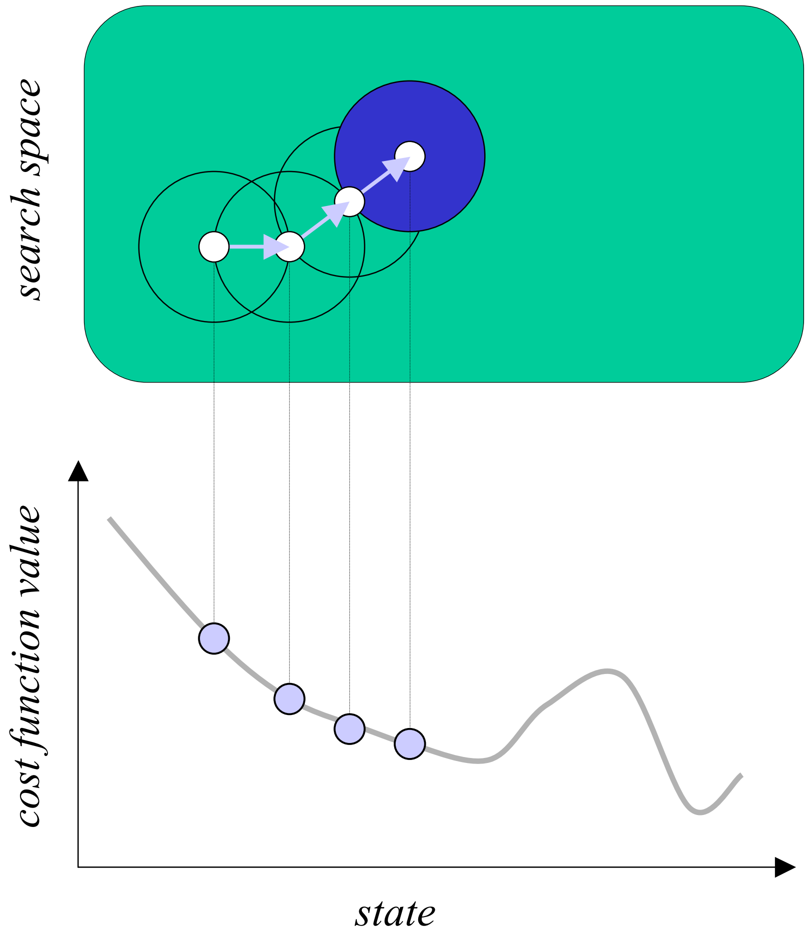
Tabu search

Neighbourhood search in TS



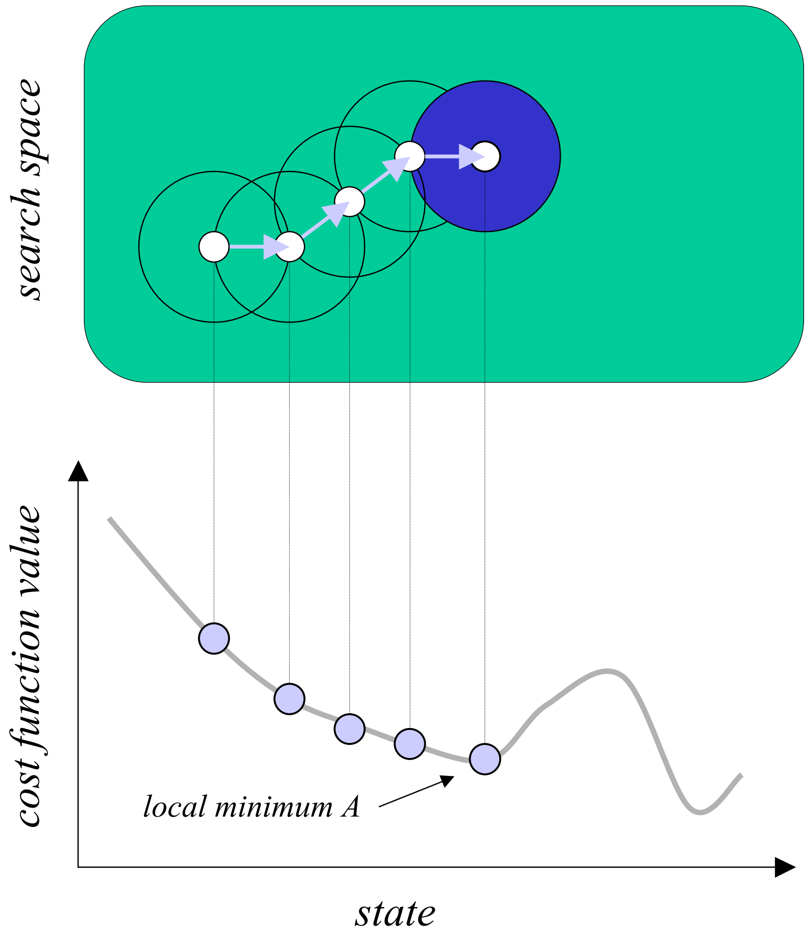
Tabu search

Neighbourhood search in TS



Tabu search

Neighbourhood search in TS

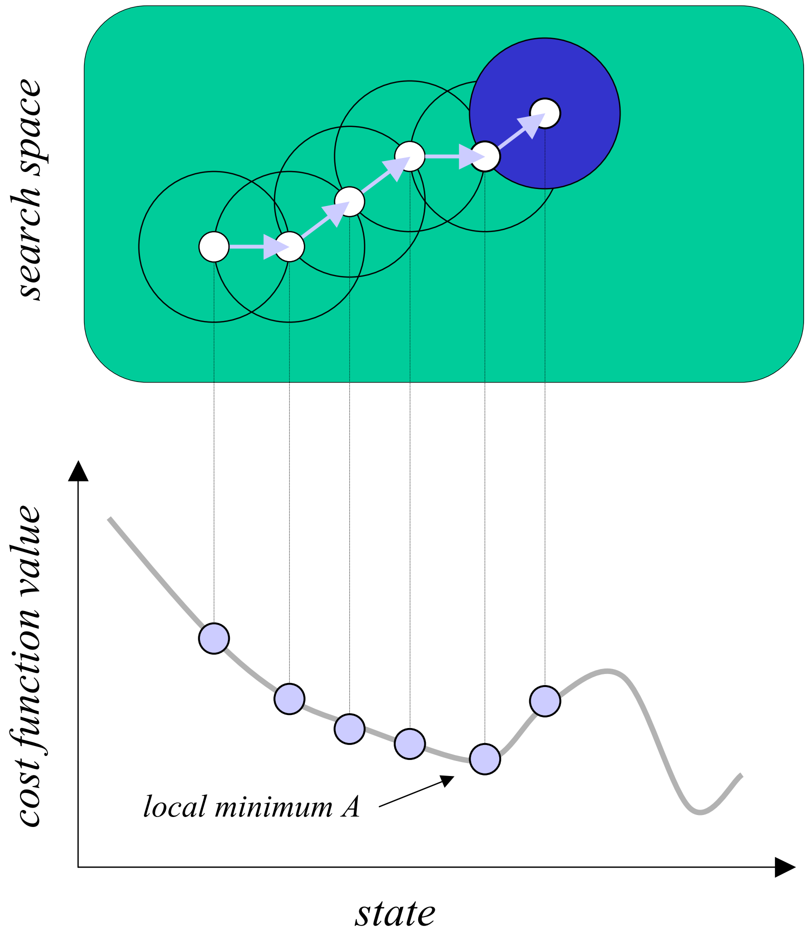


Local search

- Partendo da soluzione iniziale;
- generate altre soluzioni;
- ottenute una dall'altra (neighbourhood);
- sempre migliori;
- termina quando non sono più possibili miglioramenti.

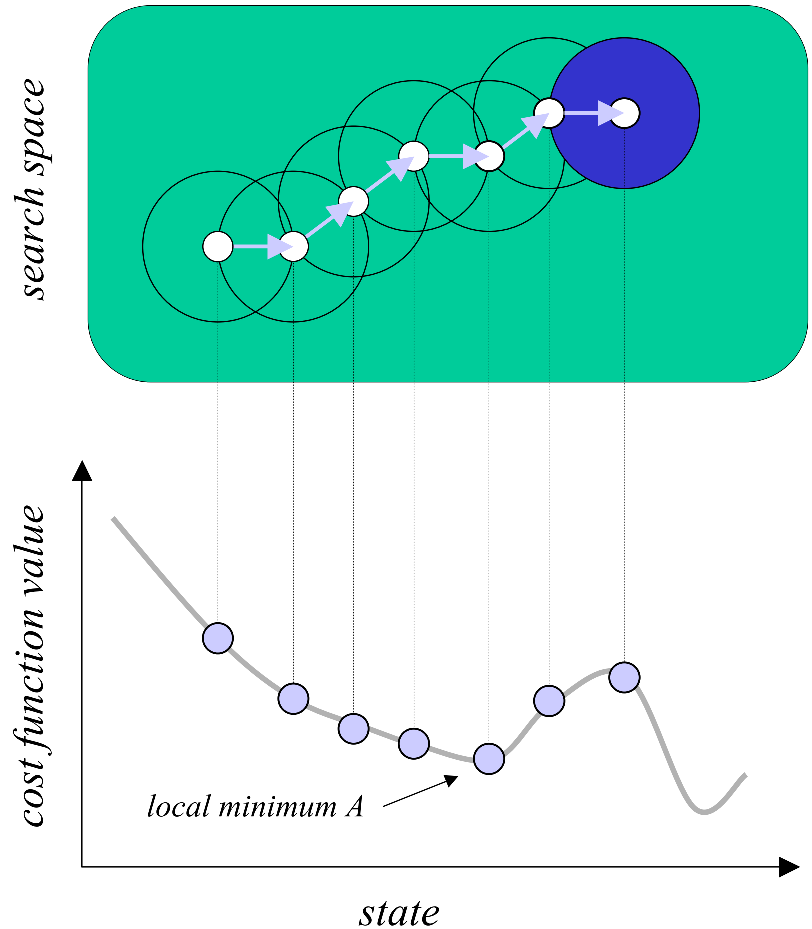
Tabu search

Neighbourhood search in TS



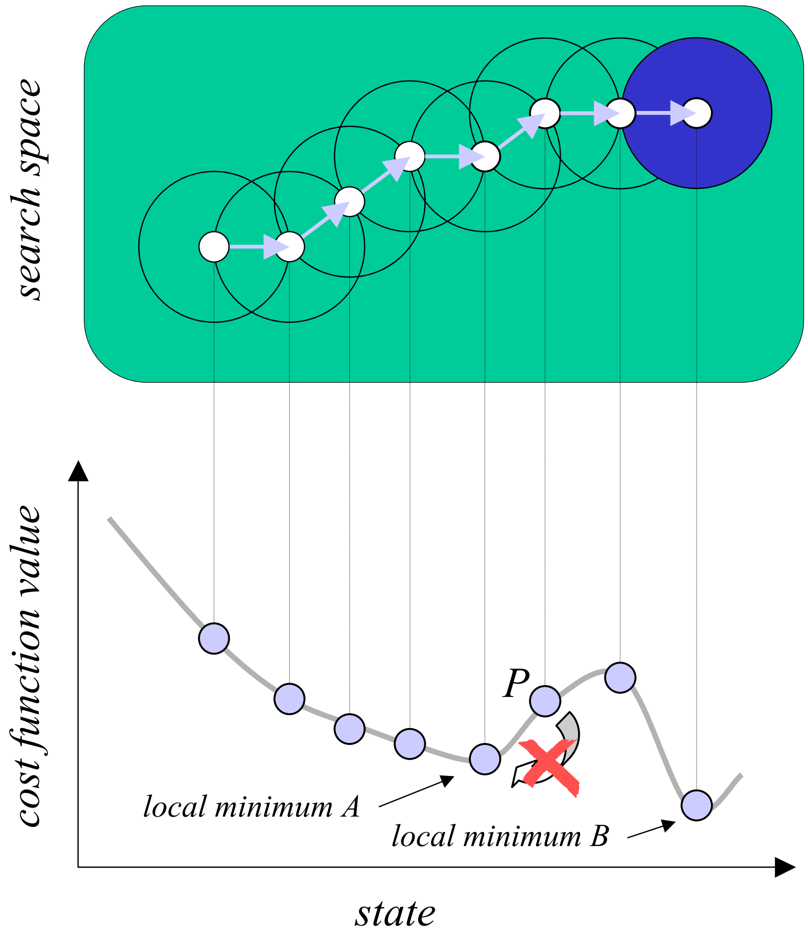
Tabu search

Neighbourhood search in TS



Tabu search

Neighbourhood search in TS



Tabu Search

- Sceglie sempre la soluzione di costo inferiore;
- anche se peggiorante.

Tabu List - TL

- Proibisce le mosse più recenti;
- memorizzando alcune informazioni.

Tabu search

1. find an initial feasible solution x;
   x^* := x;
   counter := 0; *did_not_improve* := 0;
   do
   counter ++; *did_not_improve* ++;

2. x' := best non-tabu solution in the neighbourhood of x;
   if cost(x') < cost(x^*) then
   x^* := x';
   did_not_improve := 0
   endif ;
   x := x'
   while (*counter* ≤ *max_counter*) and
   (*did_not_improve* ≤ *max_not_improve*)

Numerical results

The reference case is a 10-unit 24-hour UC problem, whose parameters of the cost functions $c_i(p_i) = a + b \cdot p_i + c \cdot p_i^2$ are published in

V. Petridis, S. Kazarlis and A. Bakirtzis, “Varying Fitness Functions in Genetic Algorithm Constrained Optimization: The Cutting Stock and Unit Commitment Problems”, IEEE Trans. on Systems, Man and Cybernetics, Part B: Cybernetics, Vol. 28, No. 5, October 1998.

The other ten UC test cases are generated from the reference with the aim to assess the influence of the various parameters of the problem on the behavior of the two different approaches.

Cases 2 and 3 are used to show the influence of the number of units

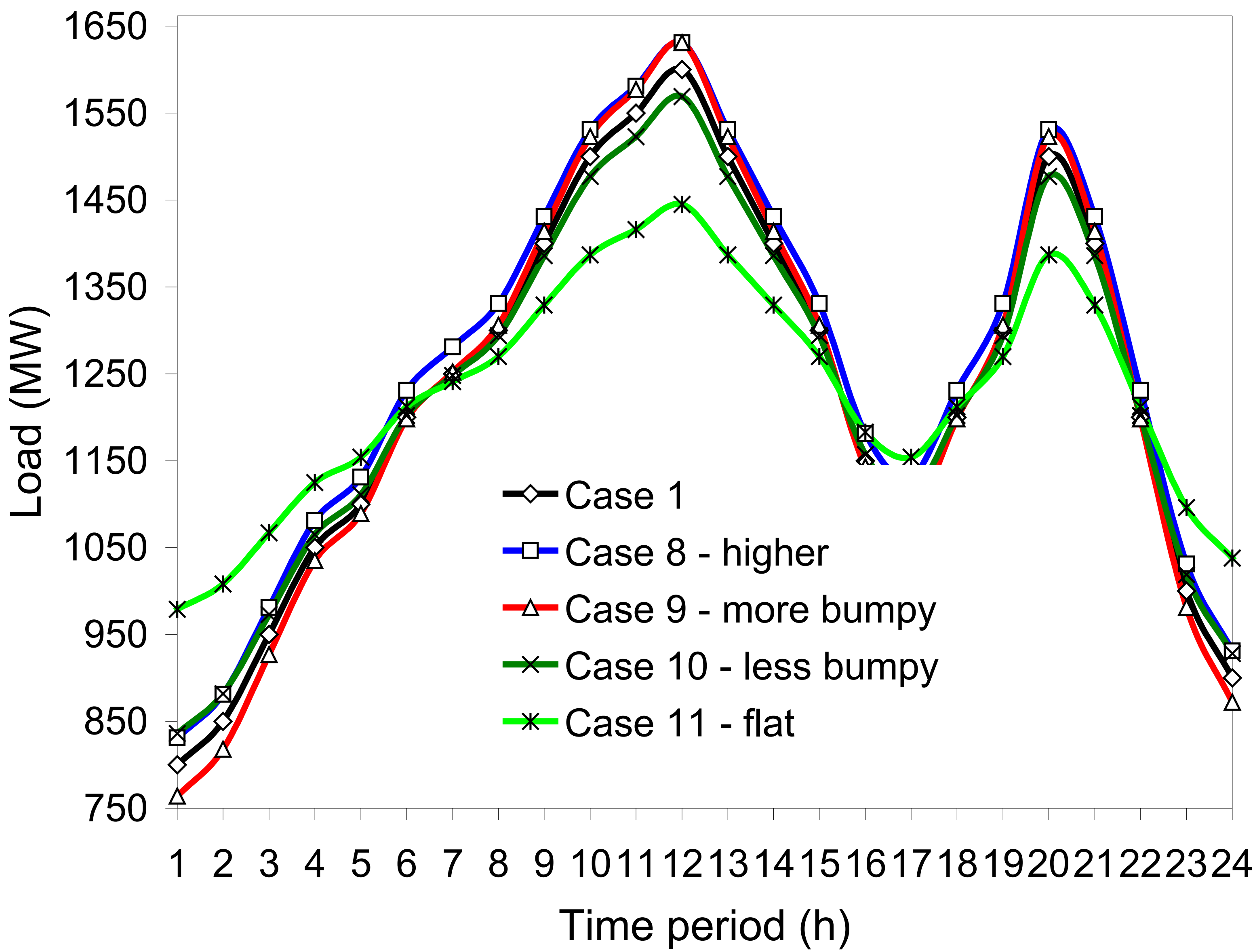
Cases 4 to 7 the influence of the size of the units

Cases 8 to 11 the influence of different demand profiles.

Numerical results

Case	No. units	Description	Load profile
1	10	Reference	Reference
2	50	Random generation of costs	Adapted from refer.
3	50	Costs equal to case 1	Adapted from refer.
4	10	Only small units (random costs)	Adapted from refer.
5	10	Only big units (random costs)	Adapted from refer.
6	50	Only small units (random costs)	Adapted from refer.
7	50	Only big units (random costs)	Adapted from refer.
8	10	Reference	Higher than refer.
9	10	Reference	More bumpy
10	10	Reference	Less bumpy
11	10	Reference	Flat

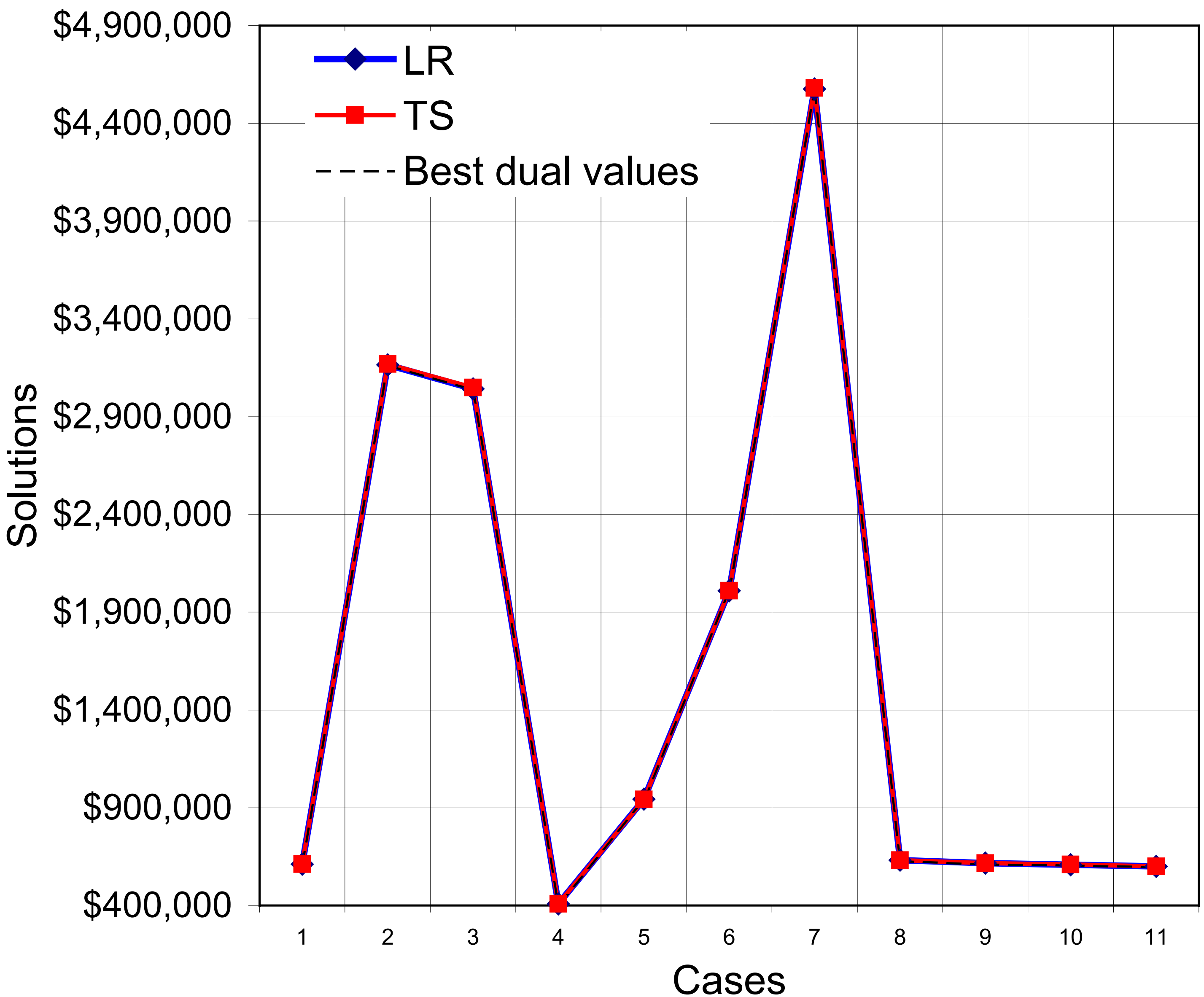
Numerical results



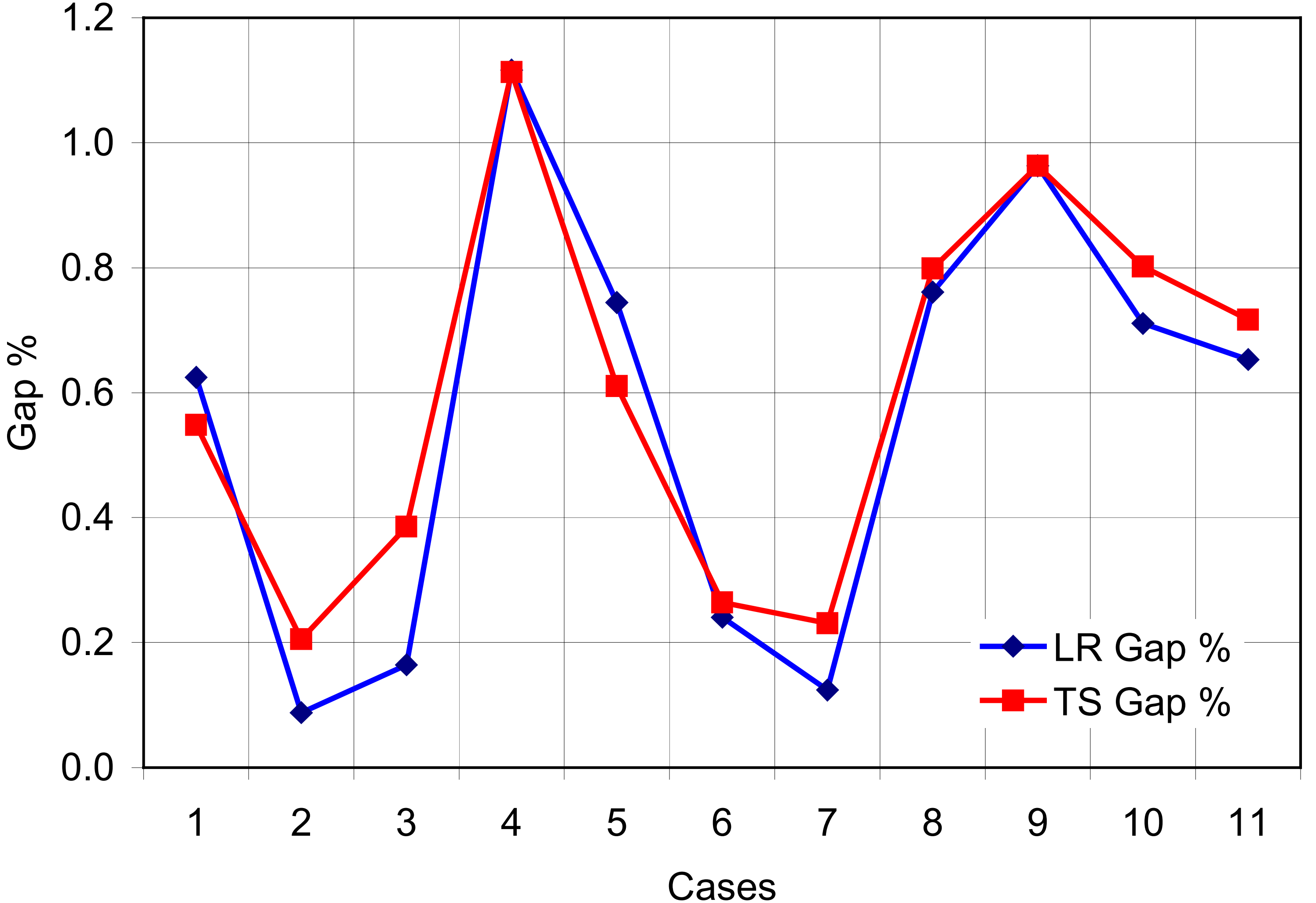
Numerical results

Case	Best dual value	Solution by L R	L R Gap (%)	Solution by TS	TS Gap (%)
1	$607,420.31	$611,214	0.625	$610,751	0.548
2	$3,162,766.76	$3,165,555	0.088	$3,169,274	0.206
3	$3,037,101.09	$3,042,094	0.164	$3,048,813	0.386
4	$403,593.70	$408,099	1.116	$408,087	1.113
5	$937,297.53	$944,275	0.744	$943,019	0.610
6	$2,004,813.58	$2,009,639	0.241	$2,010,108	0.264
7	$4,570,335.65	$4,576,011	0.124	$4,580,889	0.231
8	$626,91375	$631,683	0.761	$631,921	0.799
9	$610,335.5	$616,216	0.963	$616,214	0.963
10	$604,773.96	$609,074	0.711	$609,623	0.802
11	$596,503.07	$600,399	0.653	$600,779	0.717

Numerical results



Numerical results



Numerical results

These computational results show:

a good behavior of both approaches in finding approximate solutions for the UC;

comparable difficulties of the algorithms with respect to the various instances;

the percentage gap for the instances with 50 units is typically smaller than the one for 10 units;

as expected, Case 8 corresponding to an augmented demand profile turns out to be more costly and difficult of the reference case (instance 1). Case 9 is even more difficult, but less costly.

the quality of the solution obtained by the implemented Lagrangian relaxation seems not to be influenced by the number of units.

Conclusions

1. In this paper, a Lagrangian relaxation algorithm for the solution of UC problems has been illustrated, wherein the dual problem solution is achieved through the implementation of an improved bundle method and the feasible solution for the primal problem is computed by a heuristic procedure that exploits available hints given by the bundle algorithm. The results obtained by the LR algorithm are compared with those obtained by a Tabu Search algorithm.

2. This comparison has shown a good behaviour of both approaches in finding approximate solutions. Moreover, the analysis of the different and complementary characteristics of the two approaches suggests further research activity to obtain an integrated algorithm of them, able to provide adequate solutions of the new UC problems peculiar of competitive electricity markets.